

PROPAGATION EFFECTS IN RADIO ASTRONOMY

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Introduction

The only way to study the Universe is by means of analyzing the radiations reaching us from distant objects. The electromagnetic waves reach us after having travelled extremely large distances in time and space. As the wave propagates, the various properties of the space modifies the original signal like attenuation, absorption, change of path, change of polarization, etc. Effects like spectral absorption, Faraday rotation and magnetohydrodynamic waves are seen. The Earth's ionosphere also plays a big role in modifying some of the wave properties by effects like scintillation, Faraday rotation etc.

Here, we shall study the behavior of the electromagnetic waves in various media under different conditions. We begin with a review of the Maxwell's equations and thereby the derivation of electromagnetic wave equations

The Electromagnetic wave Equation

The wave equation can be derived from the four Maxwell's equations:

$$\left. \begin{aligned} \nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} + j\omega \mathbf{D} \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} = -j\omega \mathbf{B} \\ \nabla \cdot \mathbf{D} &= \rho \\ \nabla \cdot \mathbf{B} &= 0 \end{aligned} \right\} \dots (1)$$

Here, \mathbf{H} , \mathbf{E} , \mathbf{D} , \mathbf{B} and \mathbf{J} respectively are the magnetic field, electric field, electric flux-density, magnetic flux-density and current density. The symbols ρ and ω respectively are the electric charge density and the angular frequency.

Also $\mathbf{B} = \mu \mathbf{H}$, $\mathbf{D} = \epsilon \mathbf{E}$ and $\mathbf{J} = \sigma \mathbf{E}$, where, μ , ϵ and σ are respectively the permeability, permittivity and conductivity of the medium.

We may derive the wave equation by taking the curl of the second Maxwell's equation. The procedure is shown below:

$$\left. \begin{aligned} \nabla \times \nabla \times \mathbf{E} &= -j\omega\mu (\nabla \times \mathbf{H}) \\ &= -j\mu\omega (\mathbf{J} + j\omega \mathbf{D}) \\ &= -j\mu\omega (\sigma \mathbf{E} + j\omega \epsilon \mathbf{E}) \\ \therefore \nabla \times \nabla \times \mathbf{E} + j\mu\omega (\sigma \mathbf{E} + j\omega \epsilon \mathbf{E}) &= 0 \end{aligned} \right\} \dots (2)$$

Substituting $j\mu\omega (\sigma + j\mu\epsilon) = \gamma^2$ in above we obtain:

$$\nabla \times \nabla \times \vec{E} + \gamma^2 \mathbf{E} = 0 \quad \dots (3)$$

Here, γ is known as the *propagation constant*.

Plane Waves in Loss-less Media-I

We have seen ... $\nabla \times \nabla \times \vec{E} + \gamma^2 \vec{E} = 0 \quad \dots (3)$

Consider a plane wave propagating along the z -direction with \mathbf{E} along the y -direction. Hence E_x and E_z are zero. Hence, we may write Eq. (3) as:

$$\frac{\partial^2 E_y}{\partial z^2} - \gamma^2 E_y = 0 \quad \dots (4)$$

The solution of above is:

$$E_y = E_0 \exp(j\omega t \pm \gamma z) \quad \dots (5)$$

Here, E_0 is the amplitude of the wave and ω is its angular frequency. The propagation constant $\gamma = \alpha \pm j\beta$, where α is called the attenuation constant and β is known as the phase constant.

If the medium is loss-less, the conductivity σ is zero and so is α . Hence we may express γ as:

$$\gamma = \pm j\beta = \pm j\omega\sqrt{\mu\epsilon} \quad \dots (6)$$

Here, $\beta = 2\pi/\lambda$, where λ is the wavelength. We may further expand β as $\beta = 2\pi v/\lambda$, where v is the speed of wave propagation (m sec^{-1}) inside the medium. It is given as $v = (\mu\epsilon)^{-0.5}$, where μ and ϵ are the permeability and permittivity of the medium.

Plane Waves in Loss-less Media-II

From previous slide ...

$$E_y = E_0 \exp(j\omega t \pm \gamma z) \quad \dots (5)$$

$$\gamma = \pm j\beta = \pm j\omega\sqrt{\mu\epsilon} \quad \dots (6)$$

Since, angular frequency $\omega = 2\pi\nu$, we may express $\beta = \omega/v = \omega(\mu\epsilon)^{-0.5}$ as expressed in Eq. (6). Coming back to our problem of plane waves in loss-less medium, we may now express E_y from Eq. (5) as:

$$E_y = E_0 \exp(j\omega t \pm j\beta z) \quad \dots (7)$$

Eq. (7) represents the space z and time t variation of \mathbf{E} for a plane wave propagating along the z -direction in a loss-less media. With a negative sign on $j\beta z$, the solution is suitable for a wave traveling along the positive z -axis. Similarly, if $j\beta z$ has a positive sign, the solution is suitable for a wave traveling along the negative z -axis. Since the exponent is a pure imaginary quantity, there is no attenuation during wave propagation.

Plane Waves in Conducting Media

If the medium is conducting and if the conductivity σ is sufficient enough to fulfill the condition $\sigma \gg \omega\epsilon$, we may write the following equations:

$$\gamma^2 = j\omega\mu\sigma \quad \dots (8) \qquad \gamma = (1 + j) \sqrt{\frac{\mu\omega\sigma}{2}} \quad \dots (9)$$

The propagation constant γ (neper m^{-1}) in conducting media has both real and imaginary parts as expressed below, where α (m^{-1}) is the attenuation constant and β (rad m^{-1}) is the phase constant.

$$\gamma = \alpha + j\beta \quad \dots (10)$$

Expanding Eq. (10) using Eq. (9) :

$$\alpha + j\beta = \sqrt{\frac{\mu\omega\sigma}{2}} + j\sqrt{\frac{\mu\omega\sigma}{2}} \quad \dots (11)$$

In a conducting medium, the solution for a plane wave propagating in the z -direction with \mathbf{E} polarized in the y -direction is:

$$E_y = E_0 e^{j\omega t \pm \gamma z} = E_0 e^{-\alpha|z|} e^{j\omega t \pm \gamma z} \quad \dots (12)$$

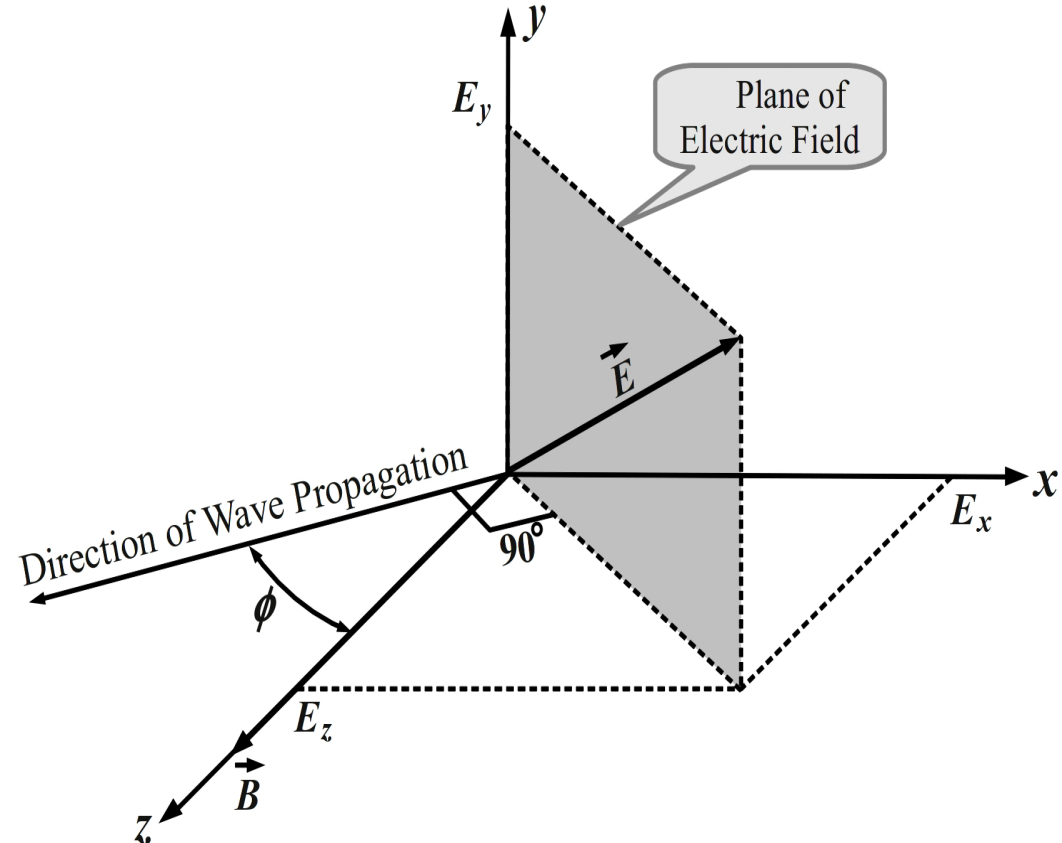
Note: Sign associated with $\alpha|z|$ in the exponent is negative, since it indicates attenuation and α is a positive quantity.

In astronomy, the product αz is termed as *optical depth*. The required distance for the wave to attenuate to $1/e$ of its original value occurs when $z = 1/\alpha$. This is marked as *unity optical depth* also known as *depth of penetration*.

Plane Waves in Ionized Media having **B** -I

Let a plane wave be propagating in an ionized medium in the presence of a steady magnetic field. The medium is assumed to have equal amount of +ve and -ve charges (plasma medium). If the plasma is cool, the collisions are unimportant. Only the interactions of electrons (-ve charges) with the wave may be considered, since the positive ions are too massive and are unable to interact significantly.

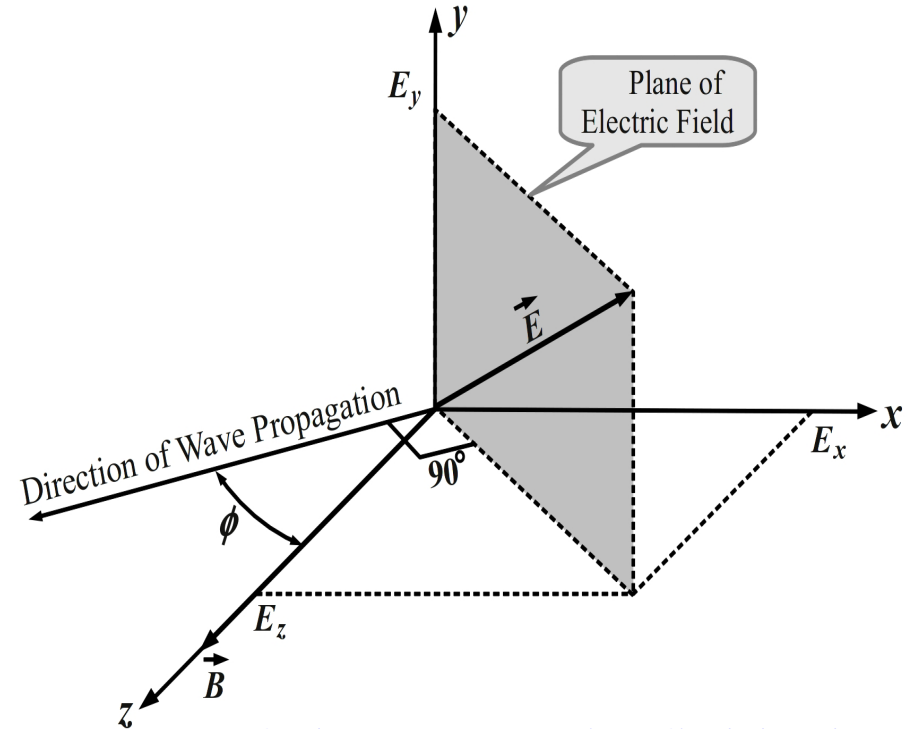
Let the ionized medium be a partially conducting medium where the condition $\sigma \gg \omega\epsilon$ is not valid. Let there be a magnetic steady field so that the medium is anisotropic, which means the propagation varies as a function of direction. The situation relatively complicated in comparison to an isotropic medium where the propagation is same in all directions.



Plane Waves in Ionized Media having **B** -II

The direction of propagating lies in x - z plane. It subtends an angle ϕ from the z -axis. Its electric field \mathbf{E} is in a plane perpendicular to the direction of propagation. A steady magnetic field \mathbf{B} is acting along the z -direction whose amplitude is B_z as expressed below.

$$|\mathbf{B}| = B_z \quad \dots (13)$$



If e is the charge of an electron, then in presence of the magnetic field, the force equations due to the magnetic field and electric field may be respectively written as shown below in Eqs. (14) and (15), where \mathbf{v} is the velocity of the electron.

$$\mathbf{F}_1 = e(\mathbf{v} \times \mathbf{B}) \quad \dots (14) \quad \mathbf{F}_2 = e\mathbf{E} \quad \dots (15)$$

The total force is $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$. It can be equated to the product of mass m and acceleration $d\mathbf{v}/dt$ of the electron using Newton's second law of motion as:

$$\mathbf{F} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = m \frac{d\mathbf{v}}{dt} \quad \dots (16)$$

Plane Waves in Ionized Media having **B** -III

From previous slide ...

$$\mathbf{F} = e (\mathbf{E} + \mathbf{v} \times \mathbf{B}) = m \frac{d\mathbf{v}}{dt} \quad \dots (16)$$

Hence, the force per unit charge will be:

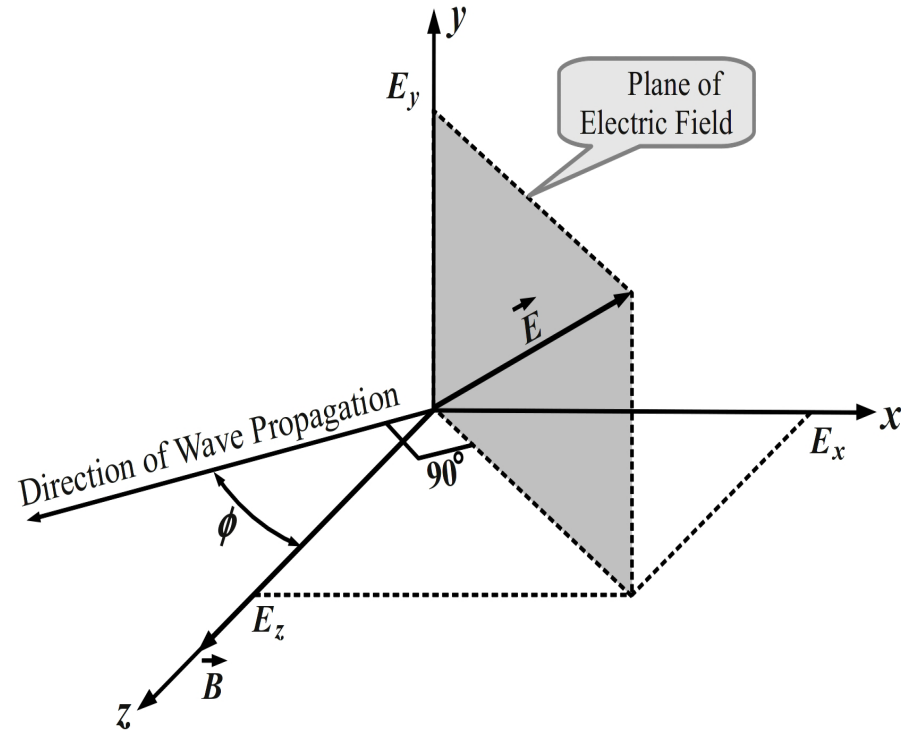
$$\frac{\mathbf{F}}{e} = \mathbf{E} + \mathbf{v} \times \mathbf{B} = \frac{m}{e} \frac{d\mathbf{v}}{dt} \quad \dots (17)$$

Expressing this vector equation in terms of rectangular component magnitudes as shown in Eqs. (18) through (20).

$$\frac{F_x}{e} = E_x + v_y B_z = \frac{m}{e} \frac{dv_x}{dt} \quad \dots (18)$$

$$\frac{F_y}{e} = E_y - v_x B_z = \frac{m}{e} \frac{dv_y}{dt} \quad \dots (19)$$

$$\frac{F_z}{e} = E_z = \frac{m}{e} \frac{dv_z}{dt} \quad \left[\text{Since, } \vec{v}_z \times \vec{B}_z = 0 \right] \dots (20)$$



Plane Waves in Ionized Media having **B** -IV

From previous slide we have ... $\frac{F_x}{e} = E_x + v_y B_z = \frac{m}{e} \frac{dv_x}{dt} \dots (18)$

$$\frac{F_y}{e} = E_y - v_x B_z = \frac{m}{e} \frac{dv_y}{dt} \dots (19) \quad \frac{F_z}{e} = E_z = \frac{m}{e} \frac{dv_z}{dt} \quad \left[\text{Since, } \vec{v}_z \times \vec{B}_z = 0 \right] \dots (20)$$

Let the motion of the charged particles (electrons) be harmonic with the wave. Hence they also oscillate at same frequency ω of the wave. If we assume any of the wave velocity components as $v = v_0 e^{j\omega t}$, then its time derivative $dv/dt = j\omega v_0 e^{j\omega t} = j\omega v$, where $\omega = 2\pi\nu$ (in rad sec⁻¹) represents the angular frequency of the wave in radians, and ν is its frequency in Hz. Substituting likewise in above equations and separating the velocity components we obtain:

$$v_x = \frac{j(m\omega/e) E_x + E_y B_z}{B_z^2 - m^2\omega^2/e^2} \dots (21)$$

$$v_y = \frac{j(m\omega/e) E_y - E_x B_z}{B_z^2 - m^2\omega^2/e^2} \dots (22)$$

$$v_z = -\frac{jeE_z}{m\omega} \dots (23)$$

It follows from the above relations that the movement of the any charged particle will follow a helical path whose axis is in coincidental with the z -direction.

Gyro Frequency of Radiation (ω_g)

Let us now consider a special case of magnetic field as shown below. It occurs when v_x and v_y are very large such that $B_z^2 - m^2\omega^2/e^2 \approx 0$ as will be obtained from Eqs. (21) and (22) respectively.

$$B_z^2 = \frac{m^2 \omega^2}{e^2} \quad \dots (24)$$

Under this condition, the effect of collisions with other particles may be neglected and we may call the angular frequency ω_g of rotation of electrons around the magnetic field as the *angular gyro-frequency*. It is measured in rad sec^{-1} . The linear gyro-frequency ν_g is given as $\nu_g = \omega_g/2\pi$. These rotation of electrons around the magnetic field causes radiations at the same frequency called radiation frequency. Re-arranging Eq. (24) we may write ω_g as:

$$\omega_g = \frac{e}{m} B_z \quad \dots (25)$$

Substituting Eq. (25) in Eqs. (21) and (22), after simplification we obtain:

$$v_x = \frac{e}{m} \frac{\omega_g E_y + j\omega E_z}{\omega_g^2 - \omega^2} \quad \dots (26)$$

$$v_y = \frac{e}{m} \frac{-\omega_g E_x + j\omega E_y}{\omega_g^2 - \omega^2} \quad \dots (27)$$

Plasma Oscillation Frequency (ω_0)-I

If the electrons are cold, it is possible to show that the charge density oscillates at the plasma frequency. It is also known as *critical frequency*. If N is the number of charged particles per unit volume, then the Maxwell's equation $\nabla \times \mathbf{H} = \mathbf{J} + j\omega\mathbf{D}$ may be expressed as:

$$\nabla \times \mathbf{H} = \mathbf{J} + j\omega\mathbf{D} = Ne\mathbf{v} + j\omega\epsilon_0\mathbf{E} \quad \dots (28)$$

The movement of the charged particles are responsible for constituting the conducting current. We may now express Eq. (28) in terms of its three rectangular components as:

$$(\nabla \times \mathbf{H})_x = j\omega\epsilon_0\vec{E}_x + Ne\vec{v}_x \quad \dots (29)$$

$$(\nabla \times \mathbf{H})_y = j\omega\epsilon_0\vec{E}_y + Ne\vec{v}_y \quad \dots (30)$$

$$(\nabla \times \mathbf{H})_z = j\omega\epsilon_0\vec{E}_z + Ne\vec{v}_z \quad \dots (31)$$

Substituting the values of v_x , v_y and v_z from Eqs. (26), (27) and (23) into above three relations we get the following three equations:

$$(\nabla \times \mathbf{H})_x = j\omega\epsilon_0\vec{E}_x \left\{ 1 + \frac{Ne^2}{\epsilon_0 m (\omega_g^2 - \omega^2)} \right\} + \left\{ \frac{Ne^2\omega_g\vec{E}_y}{\epsilon_0 m (\omega_g^2 - \omega^2)} \right\} \quad \dots (32)$$

$$(\nabla \times \mathbf{H})_y = -\frac{Ne^2\omega_g\vec{E}_x}{m (\omega_g^2 - \omega^2)} + j\omega\epsilon_0\vec{E}_y \left\{ 1 + \frac{Ne^2}{\epsilon_0 m (\omega_g^2 - \omega^2)} \right\} \quad \dots (33)$$

Plasma Oscillation Frequency (ω_0)-II

$$(\nabla \times \mathbf{H})_z = 1\vec{e}_z - j\omega\epsilon_0\vec{E}_z \left\{ 1 + \frac{Ne^2}{\epsilon_0 m\omega^2} \right\} \dots (34)$$

Here \mathbf{e}_z is a unit vector along the z -direction.

When $\omega \gg \omega_g$, all the three relations become identical. The *critical* or *plasma frequency* is attained when the following equation holds true.

$$\frac{Ne^2}{\epsilon_0 m\omega^2} = 1 \dots (35)$$

Under such condition, the refractive index and the permittivity of the medium becomes zero. If we symbolize this *critical radio frequency* (plasma frequency) as ω_0 (in rad Hz⁻¹), we may rewrite Eq. (35) as shown below:

$$\omega_0^2 = \frac{Ne^2}{\epsilon_0 m} \dots (36)$$

If we express $\omega_0 = 2\pi\nu_0$, where ν_0 is the critical frequency (in Hz), then from Eq. (36) we may express ν_0 as:

$$\nu_0 = \frac{e}{2\pi} \sqrt{\frac{N}{\epsilon_0 m}} \dots (37)$$

Here m is mass of the particle (in kg), e is the charge of a particle (in coulomb) and ϵ_0 is the permittivity of free space (8.854×10^{-12} farad m⁻¹).

Plasma Oscillation Frequency (ω_0)-III

From previous slide ...

$$\nu_0 = \frac{e}{2\pi} \sqrt{\frac{N}{\epsilon_0 m}} \quad \dots (37)$$

If the charged particles are electrons, then from Eq. (37) we get the critical or plasma frequency as shown below:

$$\nu_0 \approx 9\sqrt{N} \quad \dots (38)$$

The wave is totally reflected by the medium at or below the critical frequency. For Earth ionosphere, sometimes the critical frequency comes out to be 9 MHz.

Propagation Effects above or below ω_0 -I

From previous slides, the expressions relating plasma frequency ω_0 are...

$$\omega_0^2 = \frac{Ne^2}{\epsilon_0 m} \quad \dots (36) \quad \nu_0 = \frac{e}{2\pi} \sqrt{\frac{N}{\epsilon_0 m}} \quad \dots (37) \quad \nu_0 \approx 9\sqrt{N} \quad \dots (38)$$

Maxwell magnetic curl equation may be written as shown below, where $\bar{\epsilon}$ is the tensor permittivity whose components satisfy Eqs. (36), (37) and (38).

$$\nabla \times \mathbf{H} = j\omega\bar{\epsilon} \cdot \mathbf{E} \quad \dots (39)$$

In a generalized form we may express $\bar{\epsilon}$ as:

$$\bar{\epsilon} = \left. \begin{array}{l} \left[\begin{array}{ccc} \epsilon_{11} & -j\epsilon_{12} & \epsilon_{13} \\ j\epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{array} \right] \\ = \left[\begin{array}{ccc} \left(1 + \frac{\omega_0^2}{\omega_g^2 - \omega^2}\right) \epsilon_0 & \frac{-j\omega_0^2 \omega_g \epsilon_0}{\omega(\omega_g^2 - \omega^2)} & 0 \\ \frac{j\omega_0^2 \omega_g \epsilon_0}{\omega(\omega_g^2 - \omega^2)} & \left(1 + \frac{\omega_0^2}{\omega_g^2 - \omega^2}\right) \epsilon_0 & 0 \\ 0 & 0 & \left(1 - \frac{\omega_0^2}{\omega^2}\right) \epsilon_0 \end{array} \right] \end{array} \right\} \dots (40)$$

We now study the propagation effects in (i) absence and (ii) presence of the external magnetic field \mathbf{B} as described next.

Propagation Effects above or below ω_0 -II

Case (i): When the magnetic field is absent ($\mathbf{B} = 0$), or when the frequency is much higher than the gyro-frequency ($\omega \gg \omega_g$), Eq. (40) gets simplified as:

$$\bar{\epsilon} = \begin{bmatrix} \left(1 - \frac{\omega_0^2}{\omega^2}\right) \epsilon_0 & 0 & 0 \\ 0 & \left(1 - \frac{\omega_0^2}{\omega^2}\right) \epsilon_0 & 0 \\ 0 & 0 & \left(1 - \frac{\omega_0^2}{\omega^2}\right) \epsilon_0 \end{bmatrix} \dots (41)$$

Using this, Maxwell's magnetic curl can be expressed as:

$$\nabla \times \mathbf{H} = j\omega \left(1 - \frac{\omega_0^2}{\omega^2}\right) \epsilon_0 \mathbf{E} \dots (42)$$

In the above equation, the term within brackets is the relative permittivity ϵ_r of the medium. The refractive index can be obtained as:

$$\eta = \sqrt{\epsilon_r} = \sqrt{1 - (\omega_0/\omega)^2} = \sqrt{1 - (\nu_0/\nu)^2} \dots (43)$$

Eq. (43) tells that for frequencies much above plasma frequency (ω_0), the refractive index η becomes real and wave can propagate. However, η being less than unity, the refraction is opposite which occurs when waves enter into a denser medium. For frequencies much below ω_0 , the refractive index becomes imaginary and thus the wave cannot propagate and gets reflected.

Propagation Effects above or below ω_0 -III

We have seen that ... $\nu_0 \approx 9\sqrt{N}$... (38)

Perturbed charged particles in the form of cloud may tend to oscillate at the plasma frequency. During such plasma oscillations, it is believed that certain types of radiation are produced from the solar atmosphere or the corona. The electron density within the solar corona reduces with height. Eq. (38) also suggests that the plasma frequency also diminishes with decreasing electrons. Hence, at a given frequency ν , radiation takes place from the corona at heights where $\nu = \nu_0$. In reality, most of the radiation comes from a thin layer just above the height corresponding to $\nu = \nu_0$. These conclusions are applicable only for quiet Sun.

For a disturbed Sun like when there are solar flares, shock waves or high-velocity jets rise into the corona. The plasma oscillations of electron clouds excited by various mechanisms result relatively narrow-band radiations whose frequency drifts downwards since the disturbances rise through the corona into regions having lower electron density.

Propagation Effects above or below ω_0 -IV

Case (ii): When the magnetic field \mathbf{B} is present, we have:

$$\left. \begin{aligned} \nabla \times \mathbf{E} &= -j\omega\mu_0\mathbf{H} \\ \text{or, } \nabla \times \nabla \times \mathbf{E} &= -j\mu_0\omega(j\omega\bar{\epsilon} \cdot \mathbf{E}) = \omega^2\mu_0\bar{\epsilon} \cdot \mathbf{E} \end{aligned} \right\} \dots (44)$$

Ignoring the effect of collisions, let us choose its solution in the form shown below, where r is the distance measured in the direction of wave propagation and β is the phase constant. $E = E_0 e^{j(\omega t - \beta r)} \dots (45)$

From rigorous analysis it may be shown that when the wave propagates parallel to \mathbf{B} (longitudinal propagation), phase constant β may be expressed as:

$$\beta = \omega \sqrt{\mu_0 (\epsilon_{11} \pm \epsilon_{12})} \dots (46)$$

We see that the wave contain two opposite circularly polarized components.

If the wave propagation is perpendicular to the magnetic field (transverse propagation), the phase constant β can be shown as:

$$\beta = \omega \sqrt{\mu_0 \left(\epsilon_{11} - \frac{\epsilon_{12}^2}{\epsilon_{11}} \right)} \text{ [when } \mathbf{E} \text{ is perpendicular to } \mathbf{B}] \dots (47)$$

$$\beta = \omega \sqrt{\mu_0 \epsilon_{33}} \text{ [when } \mathbf{E} \text{ is parallel to } \mathbf{B}] \dots (48)$$

When \mathbf{E} is parallel to \mathbf{B} , propagation is same as the case when no magnetic field was present. The wave is referred to as *ordinary ray*. When \mathbf{E} is perpendicular to \mathbf{B} , the wave is referred to as *extraordinary ray*.

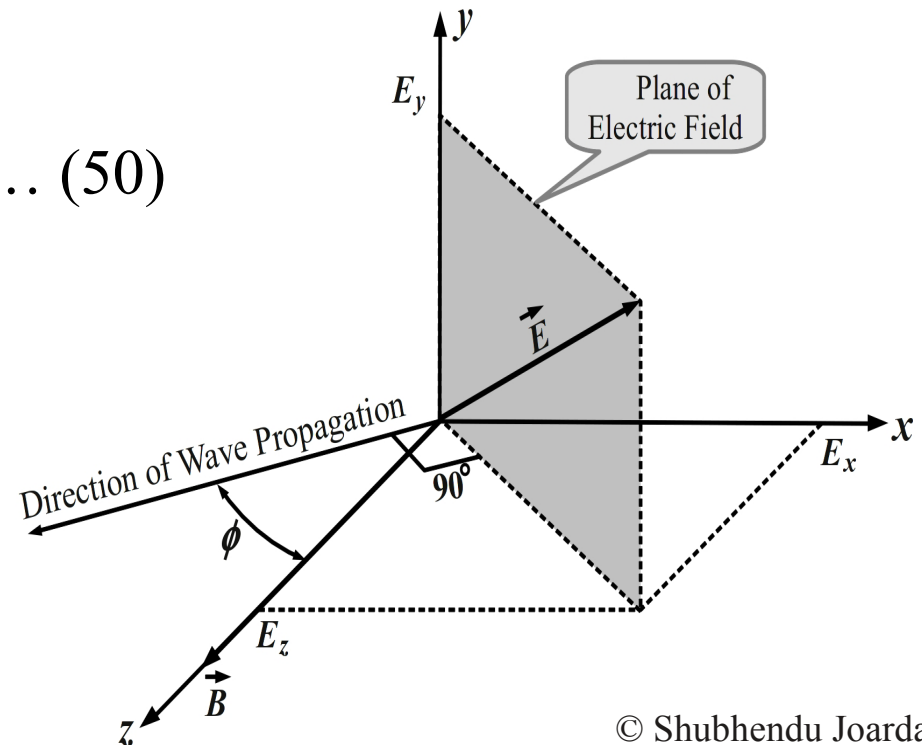
Propagation Effects above or below $\omega_0 - V$

Now we establish some more general relationships for quasi-longitudinal and quasi-transverse propagation conditions. For the former case, ϕ is sufficiently small so that the terms $\sin^2\phi$ and $\sin^4\phi$ may be ignored. For the latter case, ϕ is close to 90° and so we may ignore $\cos \phi$. Thus for a quasi-longitudinal propagation where ϕ is small, we may express β as:

$$\beta = \omega \sqrt{\mu_0 (\epsilon_{11} \pm \epsilon_{12} \cos \phi)} \quad \dots (49)$$

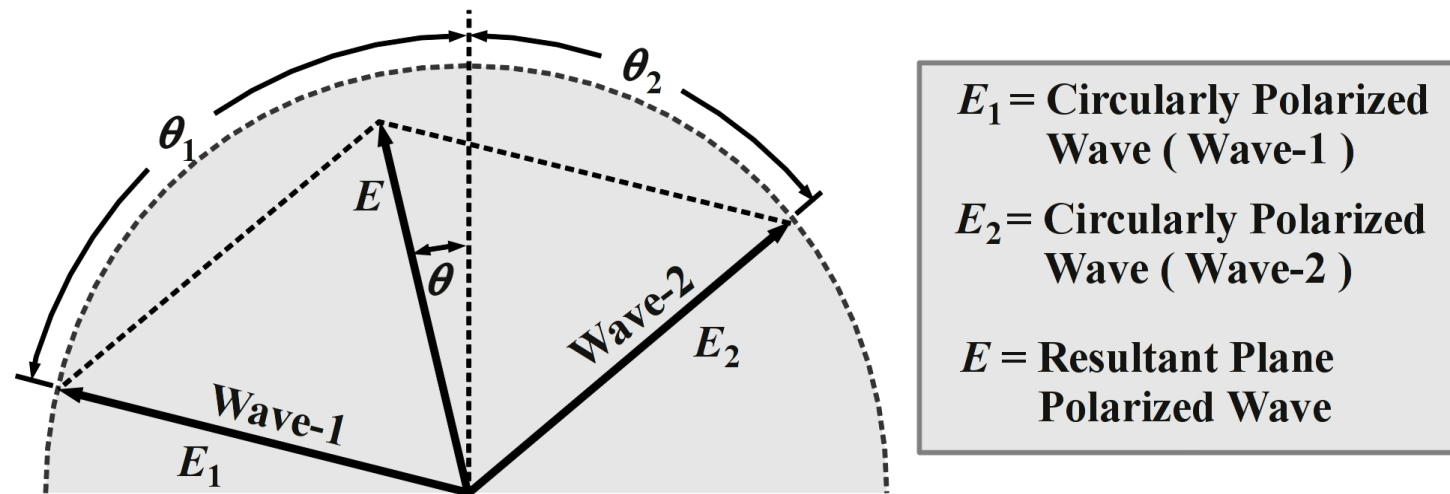
Similarly for the quasi-transverse propagation where $\phi \approx 90^\circ$, we may express β as shown below:

$$\beta = \omega \sqrt{\mu_0 \left\{ \epsilon_{11} - (1 \pm 1) \frac{\epsilon_{12}^2}{2\epsilon_{11}} \sin \phi \right\}} \quad \dots (50)$$



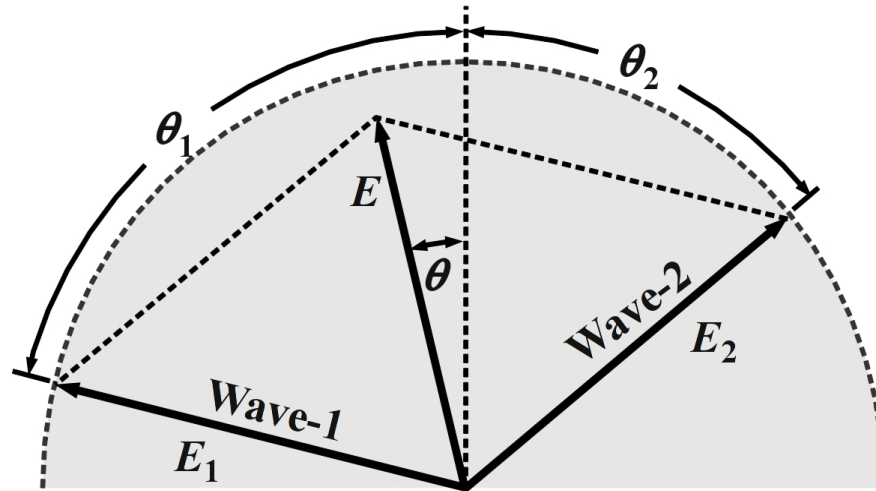
Faraday Rotation-I

In 1845, Michael Faraday observed that the plane of polarization of light passing through a crystal rotates. A linearly polarized wave may result from two circularly polarized waves having identical amplitudes but opposite rotations. If the phase constants of the two circularly polarized waves are different, the plane of polarization of the resulting linearly polarized wave rotates with its propagation.



Let us consider two circularly polarized waves as shown above, where the waves 1 and 2 rotate in opposite directions. We now consider two cases of wave propagation: (i) quasi-longitudinal and (ii) quasi-transverse as described next.

Faraday Rotation-II



E_1 = Circularly Polarized Wave (Wave-1)

E_2 = Circularly Polarized Wave (Wave-2)

E = Resultant Plane Polarized Wave

Case (i): Let the waves be traveling out in a quasi-longitudinal fashion such that ϕ is small. The elemental angular rotation over a propagation distance dr of one wave may be expressed as:

$$d\theta_1 = \beta^- dr \quad \dots (51)$$

The elemental angular rotation of the other may be expressed as:

$$d\theta_2 = \beta^+ dr \quad \dots (52)$$

The values of β^- and β^+ can be obtained from Eq. (49) with the sign in the parentheses as minus and plus respectively. The net rotation angle $d\theta$ can be shown as:

$$d\theta = \frac{\beta^- - \beta^+}{2} dr \quad \dots (53)$$

Ref. Eq.: $\beta = \omega \sqrt{\mu_0 (\epsilon_{11} \pm \epsilon_{12} \cos \phi)}$... (49)

Faraday Rotation-III

From Eq. (41) we substitute ϵ_{11} and ϵ_{12} in Eq. (49). Taking approximations for $\omega \gg \omega_g$, $\omega \gg \omega_0$ and small φ , we obtain:

$$d\theta = \frac{Ne^3\beta\lambda^2 \cos \phi}{8\pi^2 c^3 \epsilon_0 m^2} dr \quad \dots (54)$$

Ref. Eqs.:

$$\bar{\epsilon} = \begin{bmatrix} \left(1 - \frac{\omega_0^2}{\omega^2}\right) \epsilon_0 & 0 & 0 \\ 0 & \left(1 - \frac{\omega_0^2}{\omega^2}\right) \epsilon_0 & 0 \\ 0 & 0 & \left(1 - \frac{\omega_0^2}{\omega^2}\right) \epsilon_0 \end{bmatrix} \dots (41) \quad \beta = \omega \sqrt{\mu_0 (\epsilon_{11} \pm \epsilon_{12} \cos \phi)} \quad \dots (49)$$

Here, N is the number of charged particles per unit volume. The total Faraday rotation for the quasi-longitudinal case can now be obtained by integrating Eq. (54) from 0 to r as shown below, where r is the propagation distance.

$$\theta = \frac{e^3 \lambda^2}{8\pi^2 c^3 \epsilon_0 m^2} \int_0^r N B \cos \phi dr \quad \dots (55)$$

If we assume the values of \mathbf{B} and φ to be unchanging, then Eq. (55) can be re-expressed as:

$$\theta = \frac{e^3 \lambda^2 B}{8\pi^2 c^3 \epsilon_0 m^2} \int_0^r N dr \quad \dots (56)$$

With \mathbf{B} and φ known, a measurement of θ at a wavelength λ can estimate the total number of charged particles N_t within a column of cross section 1 m^2 between the source and observer using:

$$N_t = \int_0^r N dr \quad \dots (57)$$

Faraday Rotation-IV

Case (ii): For the quasi-transverse case where $\phi \approx 90^\circ$, we have:

$$d\theta = \frac{N e^4 \lambda^3 B^2 \sin^2 \phi}{32\pi^3 c^4 m^3 \epsilon_0} dr \quad \dots (58)$$

The total Faraday rotation can then be obtained as shown below:

$$\theta = \frac{e^4 \lambda^3}{32\pi^3 c^4 m^3 \epsilon_0} \int_0^r N B^2 \sin^2 \phi dr \quad \dots (59)$$

The ratio of longitudinal to transverse rotations may be determined from above equations as shown below:

$$\frac{d\theta (\text{longitudinal})}{d\theta (\text{transverse})} \simeq \frac{4\pi c m}{e B \lambda} = \frac{2m\omega}{B e} \quad \dots (60)$$

If we consider the electrons within the Earth ionosphere at a frequency of 100 MHz, and if we take the Earth's magnetic field as 5×10^{-5} weber m^{-2} , the ratio of longitudinal to transverse rotation comes to be of the order of 100.

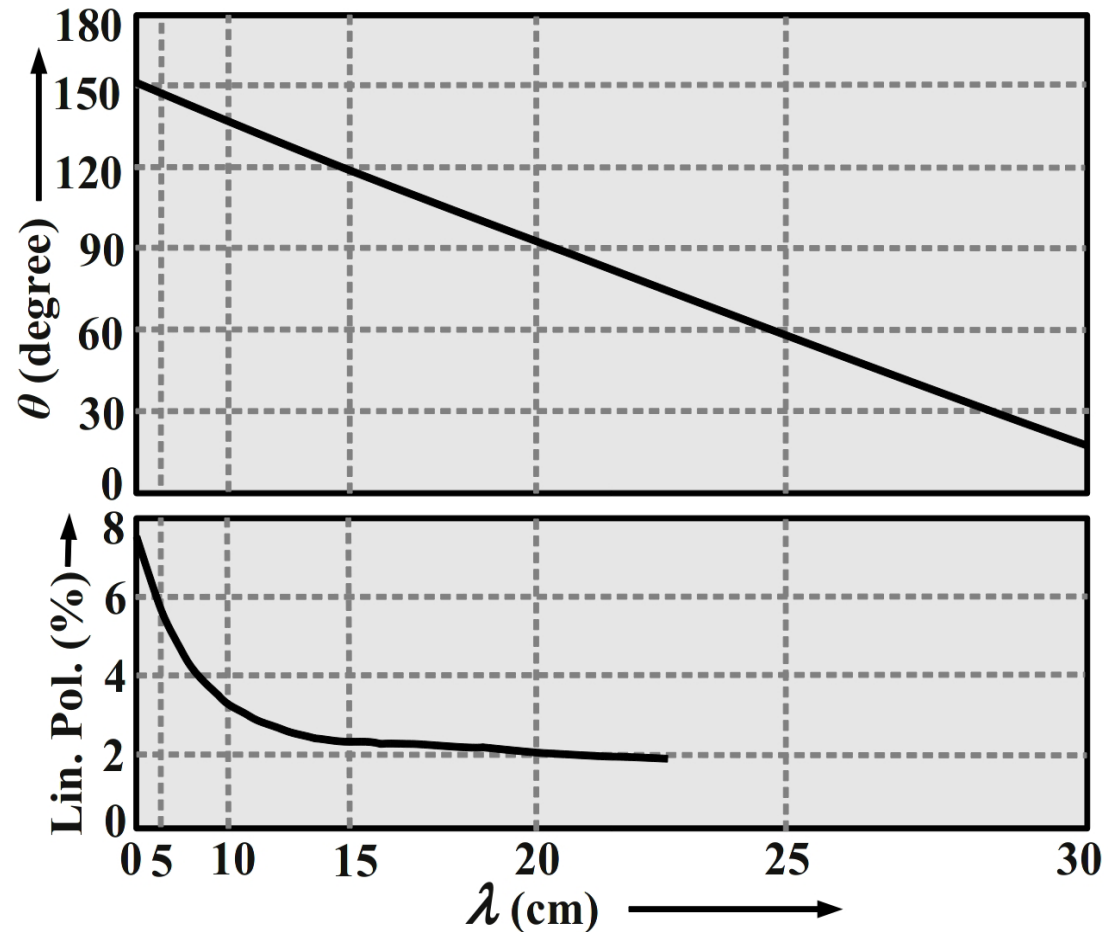
Astronomical Radio Observations-I

The magnitude of \mathbf{B} is of the order of 10^{-5} weber m^{-2} in the inter-stellar medium. As a result, when propagation is parallel to \mathbf{B} the rotation becomes dominant. This is particularly true if the frequencies chosen are higher. Thus we may treat the rotational effects entirely as in a quasi-longitudinal case.

The position angle θ of \mathbf{E} is directly proportional to wavelength's square (λ^2). Thus, θ vs. λ^2 is a straight line over a range of λ . If it is extrapolated to $\lambda = 0$, the polarization angle at the source can be determined from the slope of the line, i.e. θ/λ^2 . The rotation is positive if the direction of magnetic field is towards the observer. To avoid ambiguity, the measurements should be within a restricted range where changes in θ does not exceed $\pi/2$.

Astronomical Radio Observations-II

During early 1960s, the polarization measurements were made for Taurus A. The summarized results by Kraus are shown. The upper curve shows the variation of position angle with wavelength. The lower curve shows the percentage linear polarization as a function of wavelength. As shown, when the position angle is extrapolated at zero wavelength, the curve provides the intrinsic polarization angle (about 150°) for Taurus A (M1) which is also known as the *Crab Nebula*. The rotation measure is approximately -25 rad m^{-2} .



Position angle, and degree of linear polarization as functions of wavelength for Taurus A (M 1) or Crab Nebula. (Summarized results by Kraus).

Astronomical Radio Observations-III

We have seen that the total Faraday rotation for the quasi-longitudinal is obtained by Eq. (55) which is reproduced below, where r is the propagation distance and N is the number of charged particles per unit volume.

$$\theta = \frac{e^3 \lambda^2}{8\pi^2 c^3 \epsilon_0 m^2} \int_0^r N B \cos \phi dr \quad \dots (55)$$

If the magnetic field \mathbf{B} , angle ϕ (between the wave direction and \mathbf{B}) and the electron density are considered fixed, then Eq. (55) reduces to Eq. (61) as shown below, where Δr is the path-length in meters, is in meters, N is the number of electrons per m^3 , \mathbf{B} is in weber m^{-1} and ϕ is in radians.

$$\Delta\theta = 2.6 \times 10^{-13} N B \lambda^2 \cos \phi \Delta r \quad \text{rad} \quad \dots (61)$$

For astronomical calculations, one may prefer the number of electrons N in per cm^3 , magnetic flux-density \mathbf{B} in Gauss and path length r in pc (parsecs). The wavelength λ remains same in meter. With these units, Eq. (61) can be expressed as:

$$\Delta\theta = 8.1 \times 10^5 N B \lambda^2 \cos \phi \Delta r \quad \text{rad} \quad \dots (62)$$

For example, if we have $\mathbf{B} = 10^{-5}$ gauss, $\phi = 0$ and $\Delta r = 1,100$ pc, then for Taurus A ($\theta/\lambda^2 = 25$), using Eq. (62) we find the average electron density of the inter-stellar medium to be nearly 3×10^{-3} electrons/ cm^3 .

Astronomical Radio Observations-IV

The $\int N B \cos \varphi dr$ taken over the paths to different parts of a radio source shows clear differences. These result to differences in Faraday rotations (increasing with square of wavelength), thereby tending to depolarize the radiation which effectively decreases the degree of linear polarization. This is applicable to most of polarized radio sources. The depolarization in some sources like Cygnus A, is extremely rapid. The degree of linear polarization for Cygnus A diminishes between 8 to 1.5 percent for wavelengths between 3 and 5 cm.

Magneto Hydrodynamic Waves-I

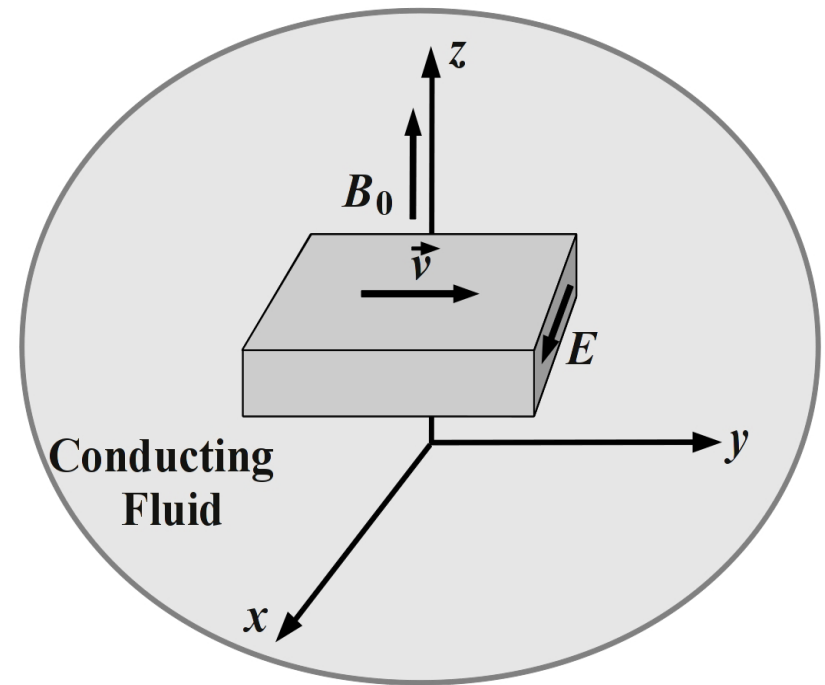
The words *magneto*, *hydro* and *dynamics* respectively stands for magnetic field, liquid and movement. The word *magneto-hydrodynamics* has been derived from them. It studies the dynamics of electrically conducting fluids. Examples of such fluids include plasmas, liquid metals, and salt water. Hannes Alfvén initiated this subject during 1960s.

We know that magnetic fields induce currents in moving conductors. Instead of a conductor if we have a moving conductive fluid in a magnetic field, currents will be induced in the liquid. These currents give rise to mechanical forces which change the state of motion of the liquid. In other words, waves may be generated within the liquid. These waves are referred as *magneto-hydrodynamic waves* or Alfvén waves.

The set of equations which may describe the magnetohydrodynamics consist usually a combination of Maxwell's equations of electromagnetism and Navier-Stokes equations of fluid dynamics. These are to be solved simultaneously by analytic or numerical methods.

Magneto Hydrodynamic Waves-II

Let a small slab shaped region of a conducting fluid be moved with a velocity \mathbf{v} in the y -direction in presence of a steady magnetic field \mathbf{B} ($|\mathbf{B}| = B_0$) applied in the z -direction. Let the extent of the slab be infinite along the y -direction. Thus an electric field \mathbf{E} will be induced in the x -direction which generates a current in the same direction. The current have its return paths through the medium (above and below the slab) which in turn produces forces. These forces tend to prevent the motion of the slab and initiates an acceleration to the fluid above and below the slab in the y -direction. Thus the original motion along y -direction initiates a motion of the fluid along z -direction which propagates as a wave.



Magneto Hydrodynamic Waves-III

Let ρ be the mass density in kg m^{-3} , p be the pressure in $\text{kg m}^{-1} \text{sec}^{-2}$ and \mathbf{G} be the acceleration in m sec^{-2} involving non-electromagnetic forces. The electrodynamic relations are listed as:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad \dots (63)$$

Maxwell Eqn.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \dots (64)$$

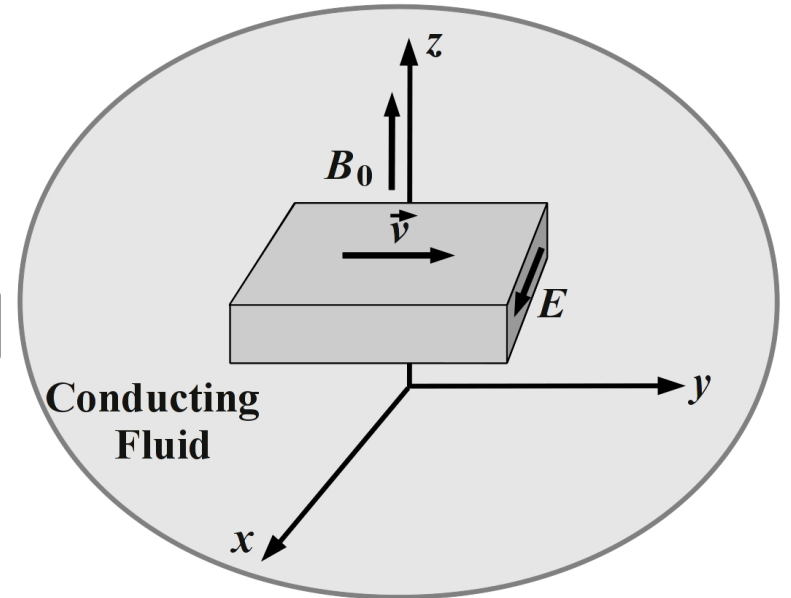
Maxwell Eqn.

$$\mathbf{J} = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad \dots (65)$$

Current density \mathbf{J} in fluid. It is sum of induced current density $\sigma(\mathbf{v} \times \mathbf{B})$ due to slab motion at velocity \mathbf{v} and conduction current density $\sigma\mathbf{E}$.

$$\mathbf{B} = \mu\mathbf{H} \quad \dots (66)$$

Characterizes the medium.



The hydrodynamic relation (dimension is acceleration) is:

$$\frac{d\mathbf{v}}{dt} = \mathbf{G} + \frac{1}{\rho} (\mathbf{J} \times \mathbf{B} - \nabla p) \quad \dots (67)$$

We have to relate the electrodynamic relations [Eqs. (62) through (66)] with the hydrodynamic relation [Eq. (67)] which is described next.

Magneto Hydrodynamic Waves-IV

Let us consider the case of plane waves in a fluid of constant density (incompressible fluid). Rearranging Eq. (65) as:

$$\mathbf{E} = \frac{\mathbf{J}}{\sigma} - \mathbf{v} \times \mathbf{B} \quad \dots (68)$$

From the geometry, we have $J_y = J_z = 0$. Thus, we may express the electric field components as:

$$E_x = \frac{J_x}{\sigma} - v_y B_0 \quad \dots (69) \quad E_y = E_z = 0 \quad \dots (70)$$

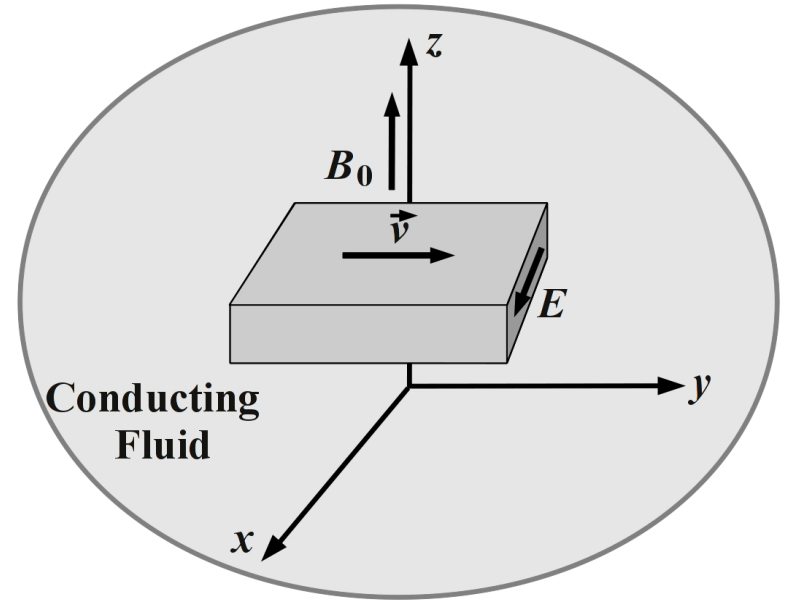
Using Eqs. (64) and (66) we obtain: $\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t} \quad \dots (71)$

Substituting the value of E_x from Eq. (69) in Eq. (71) we obtain:

$$\mu \frac{\partial H_y}{\partial t} = B_0 \frac{\partial v_y}{\partial z} - \frac{1}{\sigma} \frac{\partial J_x}{\partial z} \quad \dots (72)$$

Since $\mathbf{G} \approx 0$, from Eq. (65) we obtain:

$$\frac{\partial v_y}{\partial t} = -\frac{1}{\rho} J_x B_0 \frac{\partial J_x}{\partial z} \quad \dots (73)$$



Magneto Hydrodynamic Waves-V

It may be assumed that ∇p has no component perpendicular to z . From Eqs. (63) and (73) we obtain:

$$\frac{\partial v_y}{\partial t} = \frac{B_0}{\rho} \frac{\partial H_y}{\partial z} \quad \dots (74)$$

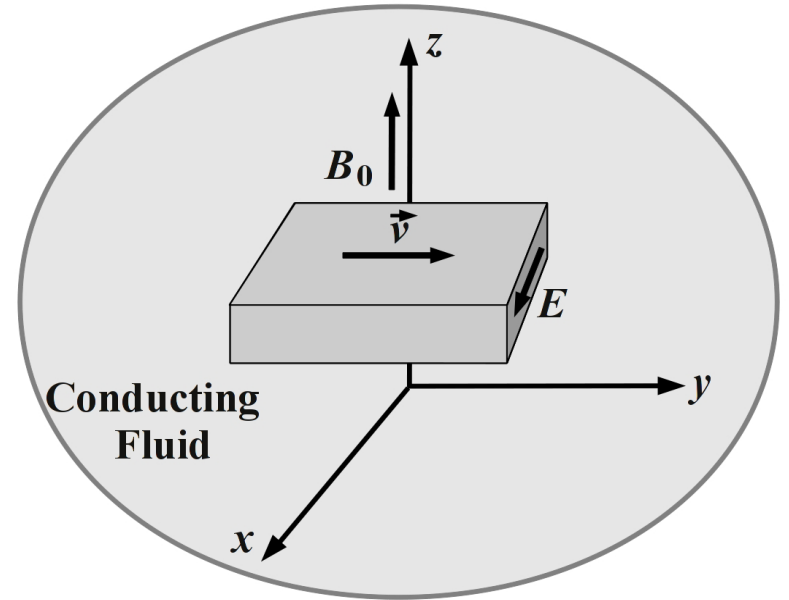
Eliminating v_y between Eqs. (72) and (74) we achieve the equation for wave propagating in the z -direction as:

$$\frac{\partial^2 H_y}{\partial t^2} = \frac{B_0^2}{\mu\rho} \frac{\partial^2 H_y}{\partial z^2} + \frac{1}{\mu\sigma} \frac{\partial^2 H_y}{\partial t \partial z^2} \quad \dots (75)$$

We can ignore the last term of Eq. (75) if the conductivity is very high. The velocity of the wave can then be expressed as:

$$v = \frac{B_0}{\sqrt{\mu\rho}} \quad \dots (76)$$

The speed v obtained from the above expression is known as the **Alfvén velocity** when expressed as a vector. Here, v is expressed in units of m sec^{-1} , B_0 is in weber m^{-2} , μ is in henry m^{-1} and ρ is in kg m^{-3} .



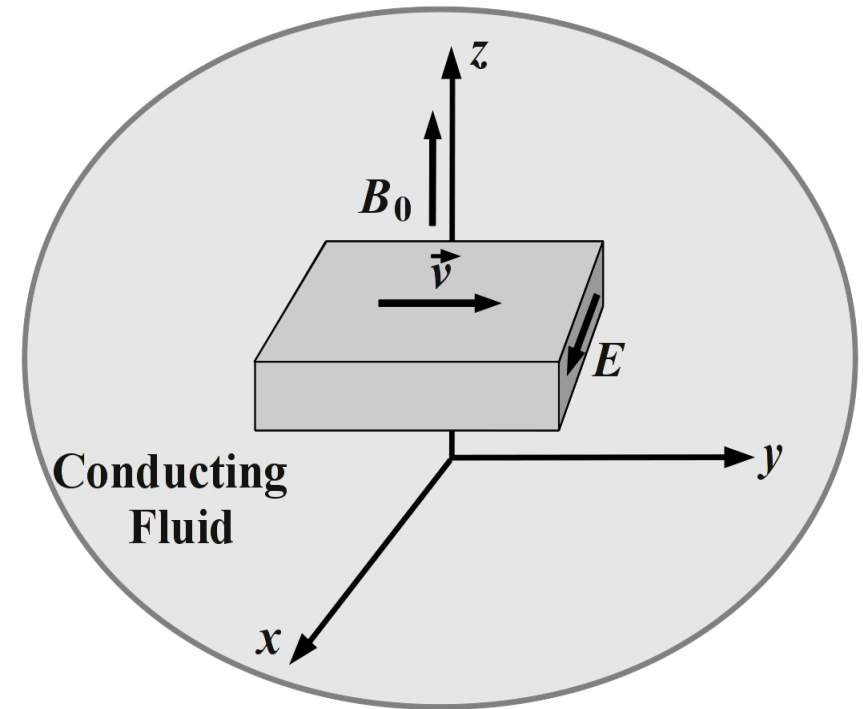
Magneto Hydrodynamic Waves-VI

With Alfvén assumptions, we now consider that the medium has infinite conductivity such that the magnetic field lines are frozen in. Thus, both the field and the medium move together and the field lines may be treated as stretched elastic strings. We may write the D'Alembert's wave equation for wave motion on a stretched string towards the z -direction as expressed below, where S is the tension of force in kg m sec^{-2} , y is the transverse displacement in meter and m is the mass per unit length in kg m^{-1} .

$$\frac{\partial^2 y}{\partial t^2} = \frac{S}{m} \frac{\partial^2 y}{\partial z^2} \dots (77)$$

Analyzing the dimensions of Eq. (77), we find that the factor S/m is equal to the square of speed v of the wave as expressed below:

$$v = \sqrt{\frac{S}{m}} \dots (78)$$

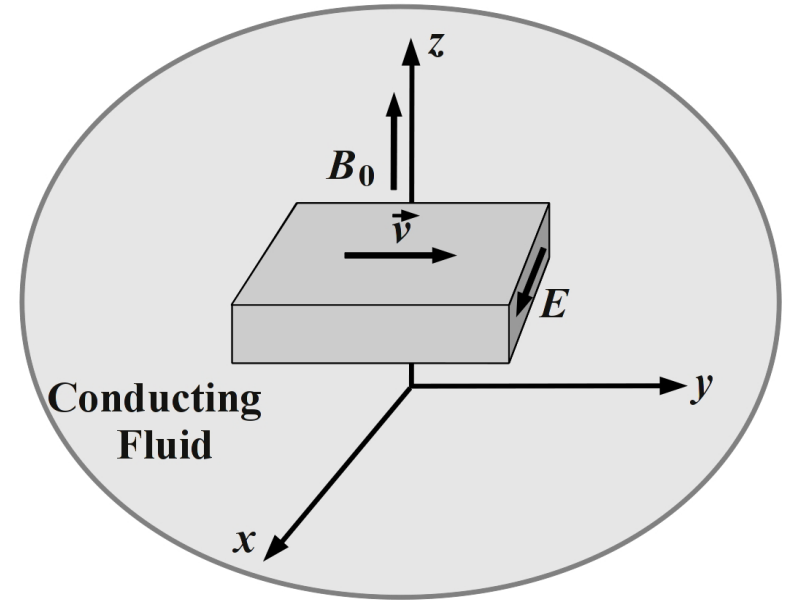


Magneto Hydrodynamic Waves-VII

Let S be the force per weber of flux in case of magnetic field, and let m be the mass per unit length per weber of flux. We thus realize the following two equations are same as found in Eq. (76).

$$\left. \begin{aligned} m &= \frac{\text{kg}}{\text{meter weber}} \\ &= \frac{\text{kg}}{\text{meter}^3 \frac{\text{weber}}{\text{meter}^2}} \\ \text{or, } m &= \frac{\rho}{B} \end{aligned} \right\} \dots (79)$$

$$v = \sqrt{\frac{H B}{\rho}} = \sqrt{\frac{B^2}{\mu \rho}} \quad [\text{m sec}^{-1}] \quad \dots (80)$$



To evaluate Eq. (76) for a magneto-hydrodynamic wave in the Sun's photosphere, we may choose (i) $B_0 = 0.1$ weber m^{-2} and (ii) $\mu = \mu_0 = 4\pi \times 10^{-7}$ henry m^{-1} . The velocity thus obtain is 6.3 km sec^{-1} . Since the density of the medium is much less in the solar corona at about 1 solar radius above the photosphere, the velocity is very close to the velocity of light.

Ionosphere on Radio Observations-I

We now apply some of the concepts developed earlier on observation of sources using radio telescopes. The Earth ionosphere plays an unwanted role in corrupting the observed data, especially below 1 GHz. In Earth ionosphere and the outer space, a gas may be in an ionized state over a long period of time. The radio waves are subject to changes when they pass through these plasma. The refractive index η of a cold neutral plasma is shown below, where ν_0 is the plasma frequency and ν is the frequency of observation.

$$\eta(\nu) = \sqrt{1 - \frac{\nu_0^2}{\nu^2}} \quad \dots (81)$$

The plasma frequency ν_0 is expressed below, where e is the charge of an electron, m_e is its mass, ϵ_0 is the permittivity of vacuum and N is the electron density (also see Eq. (37)).

$$\nu_0 = \frac{1}{2\pi} \sqrt{\frac{Ne^2}{\epsilon_0 m_e}} \approx 9\sqrt{N} \quad \dots (82)$$

The refractive index η of the medium becomes imaginary at frequencies below ν_0 . Under such conditions, most of the energy within the wave is reflected back. Some of it leaks through the medium which gets attenuated exponentially with distance and unable to cross the medium.

Ionosphere on Radio Observations-II

Electron density of the Earth ionosphere varies within 10^4 to 10^5 cm^{-3} . The corresponding ionosphere plasma frequency ν_0 also varies within the range 1 - 10 MHz. At such low frequencies, radio waves from outer space are unable to reach the Earth's surface and can only be received by space based telescopes.

Material filling the solar system through which all planets, asteroids and comets etc. move is known as *inter-planetary medium* (IPM). The plasma within the IPM and at the Earth's location has an electron density typically 0.03 cm^{-3} for which the cut off frequency is roughly 1 kHz. Such low frequency waves arriving from stars, galaxies and other objects are more or less impossible to observe even with the aid of space crafts. This is because the IPM and the inter-stellar medium (ISM) severely attenuates them.

The dispersion relationship in a cold plasma may be written as $c^2 k^2 = \omega^2 - \omega_0^2$, where c is the speed of light and k is wave-number (same as β). We may approximate the magnitude of phase velocity v_{ph} as:

$$v_{\text{ph}} = \frac{\omega}{k} = \frac{c}{\eta} \approx c \left(1 + \frac{1}{2} \frac{\nu_0^2}{\nu^2} \right) \dots (83)$$

Assuming $\nu \gg \nu_0$, the magnitude of group velocity v_{gr} is given as:

$$v_{\text{gr}} = \frac{d\omega}{dk} = c\eta \approx c \left(1 - \frac{1}{2} \frac{\nu_0^2}{\nu^2} \right) \dots (84)$$

Observing through Homogeneous Plasma-I

Many types of propagation effects exist above the cut-off frequency which affect the radio waves passing through the plasma. Let a radio wave be observed through a homogeneous plasma slab having length L . In absence of the plasma, the wave is delayed by amount ΔT as shown below, where c is the speed of light.

$$\Delta T = \frac{L}{v_{gr}} - \frac{L}{c} = \frac{L}{c} \left(\frac{1}{\eta} - 1 \right) \simeq \frac{L\nu_0^2}{2c\nu^2} \dots (85)$$

The magnitude of propagation delay may also be expressed as:

$$|\Delta T| = \frac{L}{c} \times \frac{4 \times 10^6}{\nu^2} N \dots (86)$$

The equivalent excess path length L created by the delay is equal to $c\Delta T$. Since, $(v_{gr}/c - 1)$ and $(v_{ph}/c - 1)$ differ in sign alone, the magnitude of the excess phase $[2\pi\nu (L/v_{ph} - L/c)]$ can be obtained as $\Delta\phi = 2\pi\nu\Delta T$. Since propagation delay is a function of frequency ν , different frequency components get delayed accordingly. Thus, near the slab's far end, the incident pulse gets smeared out as it propagates through the slab which is known as *dispersion*. If a magnetic field runs through the plasma, then it becomes *birefringent*. It means, the refractive index is different for right and left circularly polarized waves.

Observing through Homogeneous Plasma-II

Thus, for birefringent, the two circularly polarized components are phase shifted by different amounts due to Faraday rotation. As a result, there is a rotation of the plane of polarization. The amount of angular rotation is given as:

$$\Delta\theta = RM \lambda^2 = 0.81\lambda^2 \Delta r N B_0 \quad \dots (87)$$

Here, RM is the rotation measure. The wavelength λ is in meter, N is in cm^{-3} , B_0 is in micro Gauss and the length r of the region is considered in parsecs (see also Eq. (62) which is reproduced below).

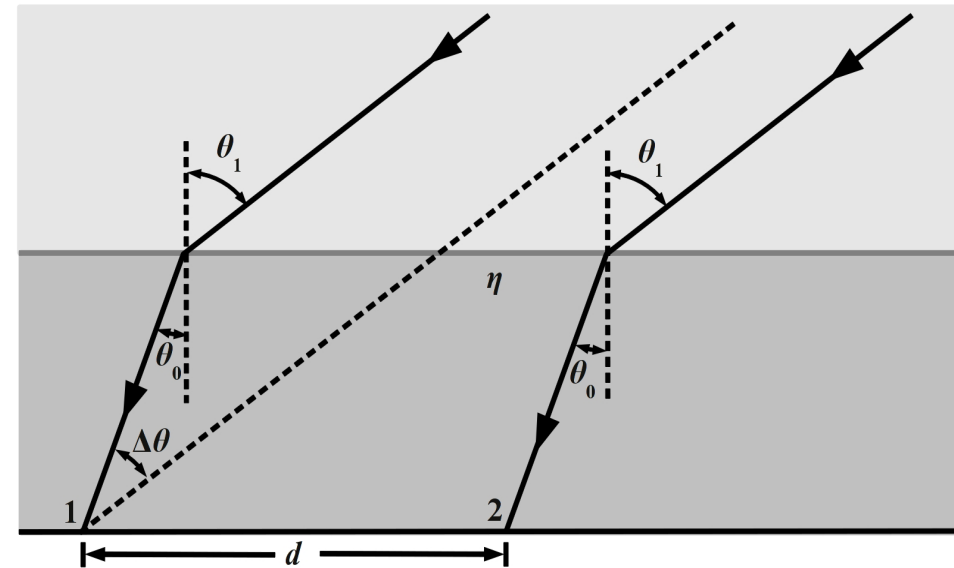
$$\Delta\theta = 8.1 \times 10^5 N B \lambda^2 \cos \phi \Delta r \text{ rad} \quad \dots (62)$$

Observing through Parallel Ionosphere

While using an interferometer, one must have a knowledge about the (i) time delay $\delta t = \Delta t_1 - \Delta t_2$ between the signals reaching the two arms, where Δt_1 and Δt_2 are the propagation delays in the two interferometer arms, and (ii) the phase difference $\delta\phi = (2\pi/\lambda)(\Delta L_1 - \Delta L_2)$ between the two signals reaching the two arms, where ΔL_1 and ΔL_2 are the excess path lengths in the two arms at a wavelength λ . In general, δt is small in comparison with the coherence bandwidth of the signal.

A parallel ionosphere with refractive index η is shown. From Snell's law we have $\sin \theta_0 = \sin \theta_1$. The observed geometric delay τ_g is given by $\tau_g = \eta d \sin \theta_0 / c$, where d is the separating distance, c is the speed of light and c/η is the magnitude of group velocity.

Applying Snell's law, we get $\tau_g = \eta d \sin \theta_0 / c$. Thus, a homogeneous plane parallel ionosphere has no net effect over the visibilities. Even if the apparent position of the source is changed with the interferometer located outside the slab, no change is observed in apparent position or phase.



Observing through Curved Ionosphere

In reality, the ionosphere is curved having a radial variation of electron density. Thus the change in apparent position and $\delta\phi$ are non-zero even outside the ionosphere. The arrival of rays from distant source effectively appear different from actual direction of arrival.

The difference between the true and the apparent directions $\Delta\theta$ is given as:

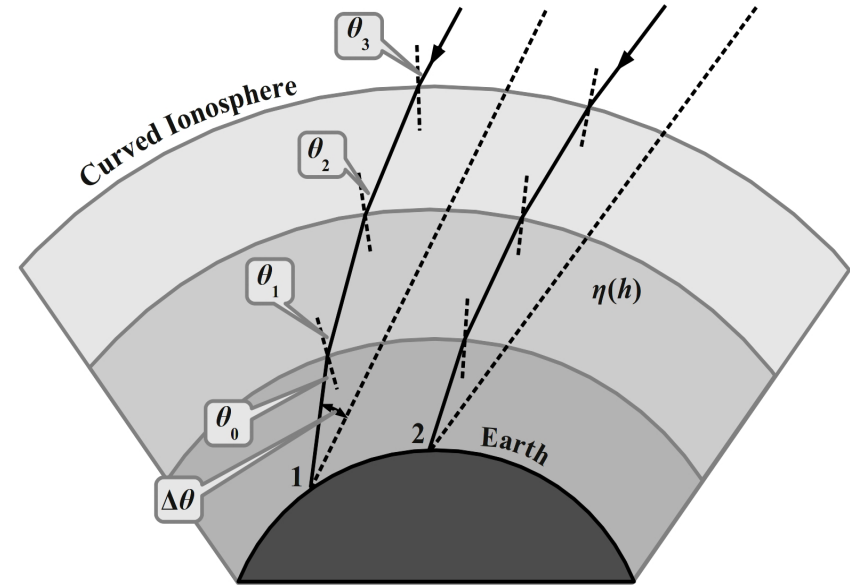
$$\Delta\theta = \frac{K \sin(\theta_0)}{r_0} \int_0^\infty \left[\frac{\alpha^2 \eta(h)}{1 - \alpha^2 \sin^2(\theta_0)} \right] dh \quad \dots (88)$$

Here, K is a constant, θ_0 is the observed zenith angle, h is the height above the Earth surface, r_0 is the radius of the Earth, and $\eta(h)$ is the refractive index at height h .

If the baseline uses u - v coordinates, the phase difference for the apparent position change of the source is:

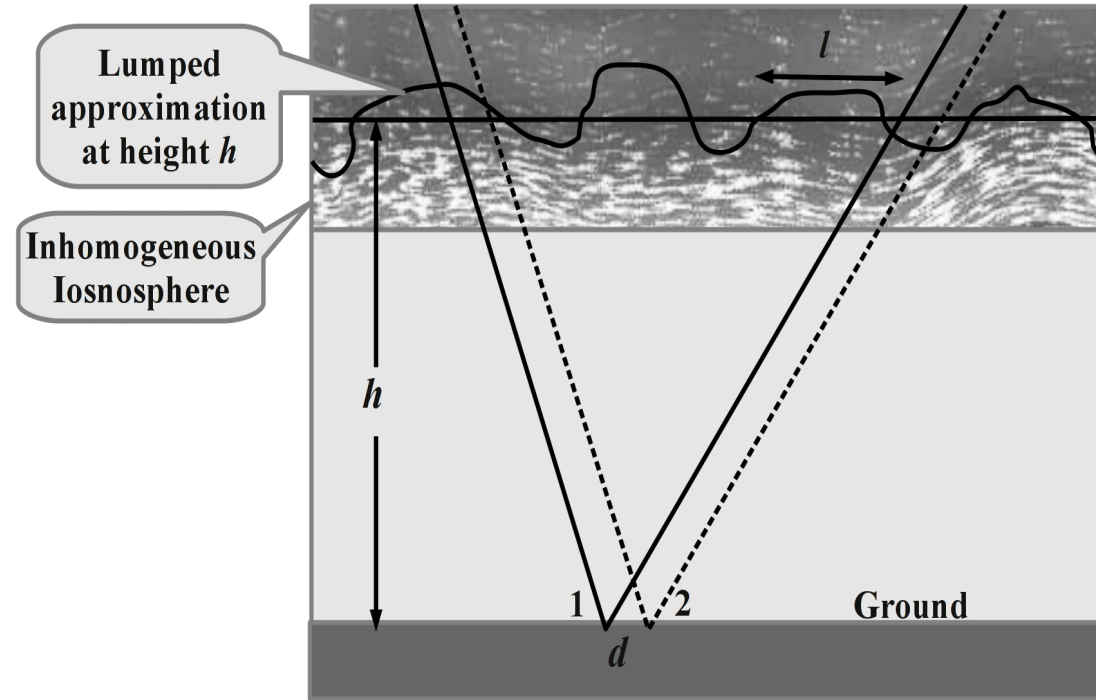
$$\Delta\phi = 2\pi (u \Delta\theta_{EW} + v \Delta\theta_{NS}) \quad \dots (89)$$

Here, $\Delta\theta_{EW}$ and $\Delta\theta_{NS}$ respectively represent the components of along East-West and North-South directions.



Observing: Nonhomogeneous Ionosphere-I

The Earth ionosphere shows density fluctuations on large time scales and lengths. Above the Earth's surface at a height h , the density fluctuation of length l corresponds to a fluctuation in an angular scale of l/h . Typically, for $l = 10$ km at a height $h = 200$ km, the angular scale (phase change) is roughly 3° . Hence an astronomer requires an unresolved source (for using it as a phase calibrator) within 3° of the radio source under observation.



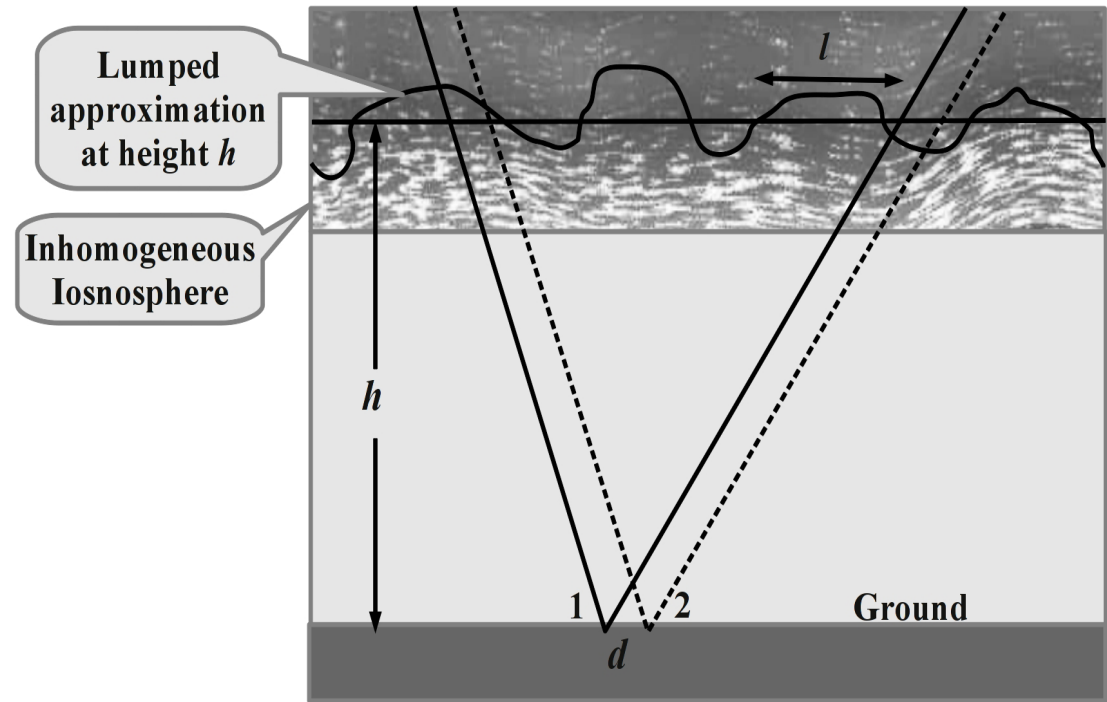
Non-homogeneous ionosphere with a short baseline d . Both interferometer arms get almost same excess phase since they look through almost same ionosphere.

The problem worsens with decrease in observation frequency since the ionospheric phase is proportional to inverse of the square of frequency. This phase may be combined with the electronic phase of the receiver system and can be solved in self-calibration, provided the excess ionospheric phase remains constant over the field of view within that period.

Observing: Nonhomogeneous Ionosphere-II

The field of view of an antenna increases inversely with frequency. The ionospheric phase variations over the field of view is known as *non isoplanaticity*. The excess ionospheric phase increases rapidly with decreasing frequency. Hence, self-calibrating algorithm can't be applied over long baselines. If the interferometer baseline length is smaller than the typical length scale of ionospheric density fluctuations, the excess phase is almost identical at both ends of the baseline.

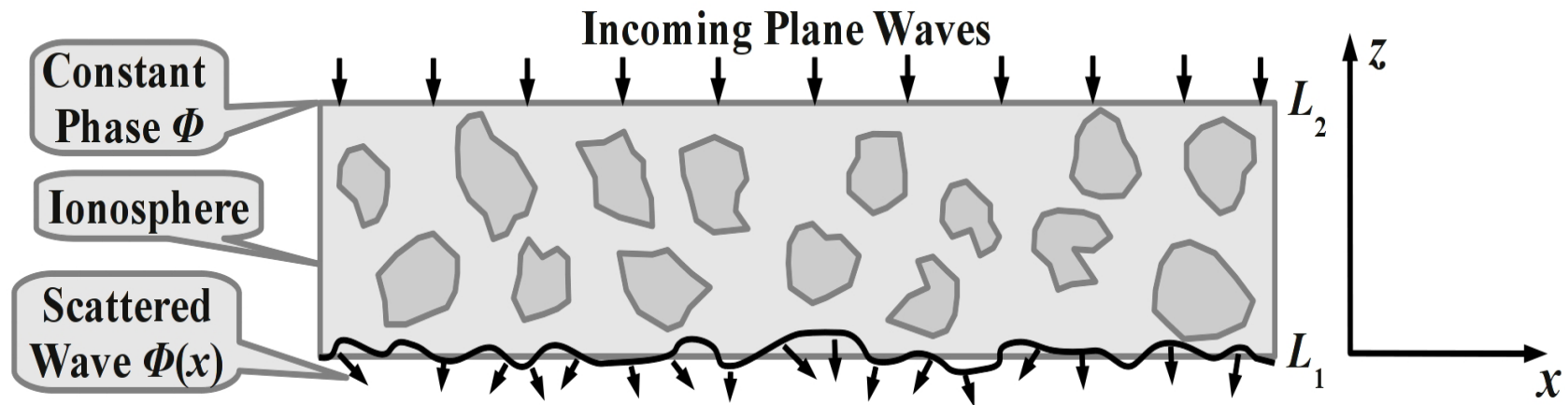
Since interferometers are sensitive only to the phase differences (between the two antenna signals), the isoplanatic assumption is still applicable. When both the baselines and the field of view are sufficiently large, the non isoplanaticity situation arises.



Non-homogeneous ionosphere with a short baseline d . Both interferometer arms get almost same excess phase since they look through almost same ionosphere.

Source Broadening from Scattering-I

The Earth ionosphere consists of plasma irregularities which affect the waves passing through them. As the waves progress, they are scattered in various directions depending on the plasma irregularities. Hence the observer sees an angular broadening of the source.



Small scale fluctuations of ionospheric electron density contribute an excess phase to radio waves propagating through it. If the change in refractive index is due to electron density fluctuation, then the excess phase is given as :

$$\phi(x) = \frac{2\pi}{\lambda} \int_{L_1}^{L_2} \Delta\eta dz = C\lambda \int_{L_1}^{L_2} \Delta N(x, z) dz \quad \dots (90)$$

Here $N(x, z)$ is the fluctuation of in electron density at the point (x, z) , λ is wavelength and C is a constant. As shown in the above figure, the integration is performed over the entire path traversed by the ray.

Source Broadening from Scattering-II

Let $\phi(x)$ be a Gaussian process having zero mean as expressed below, where σ is the standard deviation.

$$\phi(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right) \quad \dots (91)$$

The auto-correlation function $R_{\phi\phi}$ is given below, where $\phi_0 = 1/(2\pi\sigma^2)^{1/2}$ and $\rho(r) = \exp(-r^2/(2\sigma^2))$.

$$R_{\phi\phi} = \langle \phi(x)\phi(x+r) \rangle = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) = \phi_0^2 \rho(r) \quad \dots (92)$$

From Eq. (90), we find that ϕ_0 is proportional to $\lambda^2 (\Delta N)^2 L$, where L is the total path length of the ionosphere. Consider a plane wave-front produced from an extremely distant point radio source. Let the wave-front be incident on top of the ionosphere. The wave reaching the surface of the Earth would be a plane wave if the ionosphere was absent. For this case, the visibility or correlation function of the electric field is obtained as $\langle E_i(x) E_i^*(x+r) \rangle = E_i^2$, which is a constant and independent of r .

Since the ionosphere exists, different parts of the wave-front acquire different phases. Hence the emerging wave-front is not planar. If $E(x)$ is the electric field at some point within the emergent wave, we may write $E(x) = E_i e^{-j\phi x}$.

Source Broadening from Scattering-III

Since E_i is a constant, the correlation of the emergent field can be written as:

$$\langle E_i(x) E_i^*(x+r) \rangle = E_i^2 \langle e^{-j(\phi(x)-\phi(x+r))} \rangle \dots (93)$$

From statistical nature of $\phi(x)$ we can modify above as:

$$\langle E_i(x) E_i^*(x+r) \rangle = E_i^2 e^{-2\phi_0^2(1-\rho(r))} \dots (94)$$

Since $(1-\rho(r))$ increases with increasing r , thus when ϕ_0^2 is extremely large, the exponent tends to zero. Hence it may be adequate to evaluate with small values of r . This may be done by expanding $\rho(r)$ using a suitably truncated Taylor series like $\rho(r) \approx 1-r^2/2a_\phi^2$. Thus, Eq. (94) can be re-expressed as:

$$\langle E_i(x) E_i^*(x+r) \rangle = E_i^2 e^{-\phi_0^2 \frac{r^2}{\sigma^2}} \dots (95)$$

Thus we conclude that the emergent electric field has a finite coherence length. In view of van Cittert-Zernike equation it may be again concluded that the original unresolved point source has blurred out to a source of finite size. This type of blurring of point sources is known as *scatter broadening* or *angular broadening*.

Source Broadening from Scattering-IV

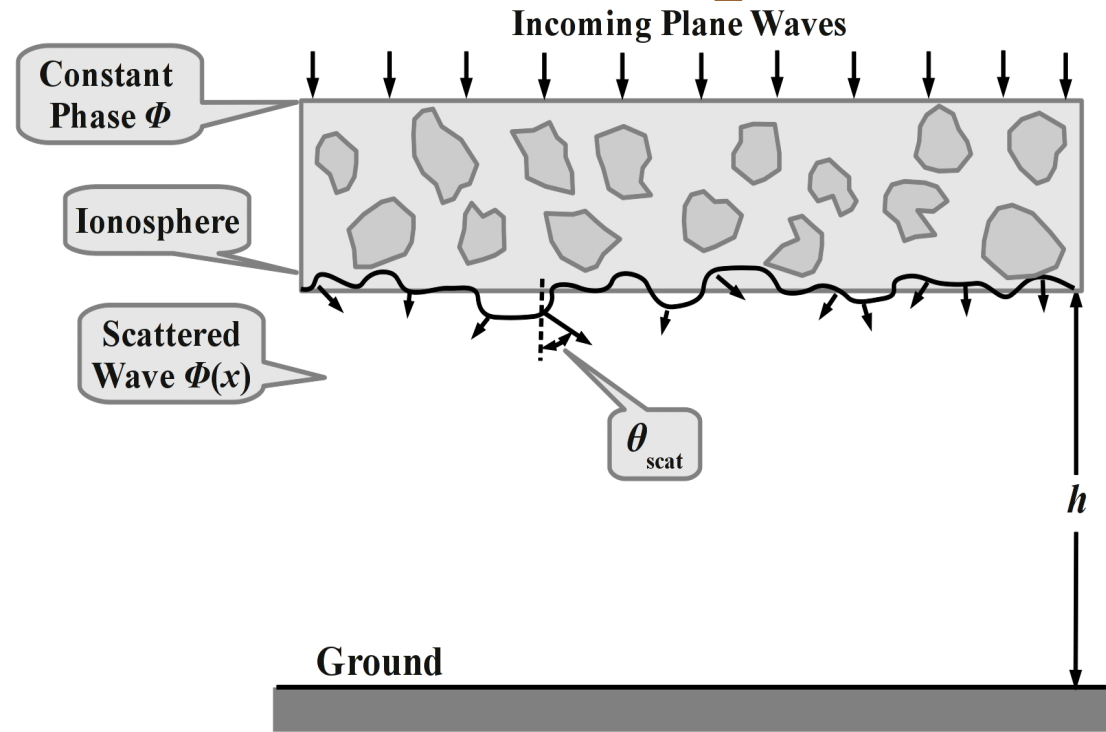
If we define $a = \sigma/\varphi_0$, the visibilities follow a Gaussian distribution and can be expressed as $\exp(ir^2)/a^2$. In other words, the characteristic angular size θ_{scat} of the scatter broadened source may be roughly shown as $\theta_{\text{scat}} \approx \lambda/a \mu \lambda^2(\Delta N)^2 L$, where θ_{scat} is called as the *scattering angle*. For small φ_0^2 , the exponent in Eq. (94) is expanded using a Taylor series as shown:

$$\left. \begin{aligned} \langle E_i(x) E_i^*(x+r) \rangle &= E_i^2 [1 - 2\phi_0^2(1 - \rho(r))] \\ &= E_i^2 \left[(1 - 2\phi_0^2) + 2\phi_0^2 e^{-\frac{r^2}{2\sigma^2}} \right] \end{aligned} \right\} \dots (96)$$

The above represents the visibilities of an unresolved core having a flux-density $E_i^2(1-2\phi_0^2)$, surrounded by a faint halo.

Scintillations produced from Ionosphere-I

The ionosphere considered so far had random density fluctuations with position, but not with time. In reality, the density varies both with position and time. Temporal variations are a result of both (i) intrinsic variations and (ii) traveling disturbances. The temporal variations of density fluctuations makes the coherence function to vary with time. This is known as *scintillation* in radio astronomy.



The scintillation could be weak or strong, depending on the height of scattering layer from the Earth and scattering angle.

The rays get scattered by an angle θ_{scat} when they pass through an irregular ionosphere. Let the scattering take place at a height h above the antennas. As shown in the figure, the scattered rays traverse a distance h before being detected. The traversed distance moved by a scattered ray is roughly $h \theta_{scat}$.

Scintillations produced from Ionosphere-II

For very small lengths compared to the coherence length a , the scattered rays (due to different irregularities) do not intersect before hitting the ground. The condition is $h\theta_{\text{scat}} < a$. That is, $h\theta_{\text{scat}} < \lambda/\theta_{\text{scat}}$, or $h\theta_{\text{scat}}^2 < \lambda$. Under these conditions, at any instant of time, the observer finds an undistorted image of the source, the position of which is shifted because of refraction. The density fluctuations vary with time causing the image to appear as wandering across the sky. The image created from a long exposure is essentially the average of many such wanderings. Hence the source appears to have scatter broadened size θ_{scat} . This effect may be corrected only by self-calibration on a time scale smaller than the image wander time scale. When $h\theta_{\text{scat}}^2 > \lambda$, the rays from different density fluctuations intersect and interfere with one another. More than one image is found by the observer and the amplitude of the received signal fluctuates with time due to interference. This is termed as amplitude scintillation. At frequencies below 500 MHz, scintillation could be very strong. On very short time scales, the source flux could vary with a factor of two or more. One cannot reliably eliminate this effect from data. Hence, observations are avoided during powerful amplitude scintillations.

Assignment Problems-I

1. An plane electromagnetic wave has its electric field aligned along x -axis and propagates along z -axis through a loss-less medium. If the angular frequency is ω and phase constant is β , write an equation representing the wave in time and space.

Hint: Make out your equation from $E_y = E_0 \exp(j\omega t \pm j\beta z)$

2. Modify the your equation for a conducting medium having an attenuation constant α .

3. How are attenuation constant and phase constant related to propagation constant? What is meant by depth of penetration?

Hint: (i) $\gamma = \alpha + j\beta$ (ii) Required distance for the wave to attenuate to $1/e$ of its original value.

4. Calculate the force produced by an electron travelling with a speed of 10^6 m/s perpendicular to a magnetic field of 5 mT. Assume the charge as 1.602×10^{-19} C.

Hint: $\mathbf{F}_1 = e(\mathbf{v} \times \mathbf{B})$

Assignment Problems-II

5. A proton travels with a velocity 10^6 m/s perpendicular to a magnetic field of 5 mT. Find the radius of curvature of the proton's path. Assume the mass of the proton as $m = 1.673 \times 10^{-27}$ kg and its charge as 1.602×10^{-19} C.

Hint: Centrifugal force $F = m\dot{v}^2/R$, where R is the radius of curvature. Balance this with $\mathbf{F} = e(\mathbf{v} \times \mathbf{B})$ and solve for R .

Ans: 20.8 m

6. An electron rotates around a magnetic field of 5 mT. Calculate the gyro-frequency given its mass as 9.10938×10^{-31} kg and charge as 1.602×10^{-19} C. What is the radiation frequency?

Hint: Use $\omega_g = (e/m)B_z$ and then convert to ν_g .

7. Calculate the plasma frequency of the ionosphere, given the number of electrons as 10^{12} per m^3 .

Hint: $\nu_0 \approx 9\sqrt{N}$

8. What is the significance of critical frequency and how is it related with plasma frequency of the ionosphere?

Assignment Problems-III

9. Explain the concept of Faraday rotation using a diagram and equations.
10. Calculate the total Faraday rotation at a frequency of 150 MHz, given the density of electrons in ionosphere as 10^{12} per m^3 , magnetic field as 10^{-5} gauss, angle between the direction of propagation and magnetic field as 30° , length of the ionosphere as 1000 km.

Hint: Calculate the wavelength and use:

$$\Delta\theta = 2.6 \times 10^{-13} N B \lambda^2 \cos \phi \Delta r \quad \text{rad}$$

11. Explain the principle of generation of magneto-hydrodynamic waves in a conducting liquid in presence of a magnetic field using a diagram.
12. Given the plasma frequency as 9 MHz, calculate the refractive index of the ionosphere at the following frequencies: (i) 150 MHz, (ii) 233 MHz, (iii) 327 MHz, (iv) 610 MHz, and (v) 1420 MHz.

Hint: Use

$$\eta(\nu) = \sqrt{1 - \frac{\nu_0^2}{\nu^2}}$$

Assignment Problems-IV

13. Calculate the group and phase velocities for problem no. 10 at all the given frequencies in problem no. 12.

Hint: Use

$$v_{\text{ph}} = \frac{\omega}{k} = \frac{c}{\eta} \approx c \left(1 + \frac{1}{2} \frac{\nu_0^2}{\nu^2} \right) \quad v_{\text{gr}} = \frac{d\omega}{dk} = c\eta \approx c \left(1 - \frac{1}{2} \frac{\nu_0^2}{\nu^2} \right)$$

14. If the ionosphere is homogeneous and parallel is there any net effect over the visibilities? If not why?

15. What are the effects of a curved homogeneous ionosphere over the visibilities if the interferometer baselines are long?

16. Why phase calibrators are used within 3° of target source? What happens when phase calibrators are at larger angles from the target Source?

17. What are the reasons for broadening of the source? Explain with a diagram.

18. How are scintillations caused? Is there any remedy against scintillation?

THANK YOU